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Quantifying fluvial non linearity and finding self organized criticality? Insights from simulations of river basin evolution

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Abstract

A numerical study was undertaken to investigate non linearity and the potential for self-organized criticality (SOC) in the evolution of river basins. Twenty-three simulations were carried out, using the authors' CAESAR landscape evolution model, in which the magnitude of storm events, variability of storm events, sediment heterogeneity, sources of sediment supply, and catchment morphology are systematically varied to evaluate their importance as possible drivers for non linear behavior and SOC.

Temporal fluctuations in simulated sediment yield show notable non linear behavior. Storm magnitude and occurrence of landslides appear to have little impact on variability of the sediment yield, when compared to the impacts of sediment heterogeneity, rainfall variability and catchment morphology. Particularly, it appears that the non linearity of sediment yields results from the manner in which the catchment processes the variable rainfall, rather than just the rainfall variability itself.

The variations in sediment yield show a power law magnitude–frequency distribution, which is a possible, but inconclusive, indicator of SOC. However, several other, more qualitative arguments can be made to support the case for SOC in these simulations. Specifically, we identify the nature of the critical state and suggest two cascade mechanisms by which the system can organize itself around this critical state. Combined, these arguments indicate that simulated evolution of river basins indeed exhibits SOC, at least with respect to sediment yield. The critical state appears to be an indicator of the connectivity of the drainage network. Thus, the simulations indicate that, unlike traditional SOC systems, the critical state of the system can vary in time, as sudden changes in drainage network connectivity may result in sudden changes in the SOC behavior of the system.
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1. Introduction

Rivers can often behave in an unpredictable, non linear and possibly chaotic way. We know that changes in rainfall can lead to changes in flooding, but the precise impacts of a flood are uncertain, as equal sized floods may cause a river to erode, to deposit, or do nothing at all. To determine how rivers react to floods, an understanding of sediment movement is important, because whilst water determines where sediment is moved, the deposited sediment then prescribes where the river can flow. It is partly this complex interaction that leads to apparently chaotic behaviour, allowing rivers to change rapidly and unpredictably through time and space — as well as changing

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independently of apparent driving variables (e.g. tectonics, climate, sediment supply). Unfortunately, as we have such a paucity of data on what causes rivers to behave chaotically, we cannot at present quantify non linear behaviour and in turn determine its significance, especially when compared to factors such as climate change.

A non linear system can be defined as one where the outputs of the system are not proportional to the inputs over the entire range of inputs (Phillips, 2003). Many examples of non linear behaviour exist within river systems, including hydraulic geometry relationships (Richards, 1973, 1976), hysteresis effects (Moog and Whiting, 1998), meander migration (Stølum, 1996; Hooke, 2003), avulsion occurrence (Bryant et al., 1995; Ashworth et al., 2004), sediment load and flood size (Moog and Whiting, 1998; Cudden and Hoey, 2003), bedload pulses (Ashmore, 1988; Gomez and Phillips, 1999), and the response of drainage basins (Schumm, 2005).

A highly non linear relationship occurs between fluxes of water and sediment in river systems, and this is a significant factor in all of the above mentioned examples. Conveniently, sediment transport also provides a metric that can be readily measured. A good example of the non linearity in sediment transport data is provided by Cudden and Hoey (2003). Studying sediment yields in a pro-glacial stream, they show how bedload fluctuates wildly during the course of daily sampling periods. Furthermore, plotting rates of bedload transport against local shear stresses reveals that, whilst a general increase of bedload occurs with shear stress, the relationship contains significant quantities of scatter. Indeed, for a shear stress of 20 Nm^{-1} rates of bedload transport can vary over nearly two orders of magnitude (40 to 4000 g^{-1} s^{-1}) (Cudden and Hoey, 2003). This difficulty in relating rates of bedload transport to flow parameters, such as shear stress or velocity, is well documented. For example, Gomez and Church (1989) compared 12 relationships between flow and sediment transport over 133 series of field and laboratory data. These revealed a wide range of scatter and a general disagreement between the equations tested, such that no single equation was ideal, and that only two performed adequately over a range of conditions (Gomez and Church, 1989). These differences in results may be caused by many factors (e.g. different sampling methods, locations, *etc.*), but are also strongly indicative of the non linear relationship between flow parameters and sediment yield.

Non linearity in a system has three major implications on the behaviour of the system. First, the system may exhibit a sensitivity to the initial conditions, as (small) changes in the inputs to the system can instigate

disproportionate changes in its outputs. Second, the behaviour of a system may exhibit emergent properties which cannot be expressed as a sum of the behaviours of its components (Phillips, 1999, 2003; Harrison, 2001). For example, physical processes of fluvial erosion and deposition can interact to build an alluvial fan as a tributary channel joins the main valley. However, the formation of the alluvial fan cannot be surmised from the processes of erosion and deposition themselves; rather, it emerges from the interactions between these processes in the prior landscape. Depending on the system and the properties considered, the emergent behaviour can be either divergent or convergent (Phillips, 1999). Divergence indicates that the system becomes more diverse over time (e.g. soil profiles, landscape dissection), whereas convergence implies the development of systematic spatial or temporal patterns (e.g. drainage network, a stable slope). If convergence occurs independent of external controls, it is an indicator of self-organization in the system. Convergences may be expressed as static equilibria, dynamic equilibria, equilibria near critical thresholds (self-organized criticality), scale-independent self-similarities in spatial patterns (fractal geometry), or scale-independent self-similarities in temporal patterns (fractal time series). The third implication of system non linearity is that, as a direct consequence of the former two, the overall large-scale, long-term behaviour of the system may be not predictable from small-scale, short-term processes (Lane and Richards, 1997). Because the emergent behaviour of a non linear system cannot be inferred from its components, a future state of the system can often only be known by direct observation at that future time — either in nature or in a model of the system.

This presents several issues for researchers studying rivers and practitioners managing them. It implies that prediction or future modelling of river behaviour could be very hard, as small changes in initial conditions can have a significant impact on system behaviour, and that we can only learn a certain amount from studying individual facets of river behaviour. This issue is all the greater given the variability of climate, and changes in the magnitude, intensity and frequency of wet weather will have significant impacts on flooding and, thus, sediment transport. In addition to improving our predictive capability, an increased understanding of what causes non linearity and its impacts will allow us to make better interpretations of what has happened in past. Correct understanding of previous fluvial events and palaeo river behaviour is a vital step in helping to understand and possibly predict and mitigate the future effects of climate change.

Another phenomenon of non linear and emergent self organised behaviour is that of Self Organised Criticality (SOC), which represent a system in dynamic equilibrium near a threshold condition (Phillips, 1999). The concept of SOC arises from Bak et al.'s (1987) study of the magnitude and frequency of landslips on a simulated sand pile, wherein they developed a simple cellular automaton model in which sand is added, grain by grain, to a surface to form a pile. When local slopes become too steep a collapse occurs, moving sediment to neighbouring cells, which too can collapse if the adjusted slopes are too steep. Bak et al. (1987, 1988) noticed that the addition of a single grain could cause a cascade of local collapses whose size could vary from a single cell to that of the whole length of the surface, and that the magnitude–frequency distribution of these cascades follows an inverse power law equation. After a collapse and with the addition of more grains of sand, the system would self organise back to this critical state where a single grain of sand could again cause a cascade of collapse. In effect, the model organised itself to a critical point through a series of positive and negative feedbacks. Bak et al. (1987, 1988) postulate that the cascading collapse behaviour observed in their model is similar to what happens when grains of sand are added to natural sand piles, where the addition of a single grain could cause an avalanche whose size could vary from a single grain to that of the whole length of the slope (e.g. Jaeger et al., 1989; Held et al., 1990; Rosendahl et al., 1993). Bak et al. (1987) referred to this tendency to evolve towards a dynamic equilibrium around a critical state as self-organized criticality (SOC).

The key features of SOC systems include:

1. The existence of a quasi-steady 'critical state' to which the system self-organizes, around which fluctuations of various magnitudes can occur.
2. An internal mechanism by which the system can reach this critical state, and by which it can recover to the critical state after a perturbation.
3. The trigger for this perturbation may be very small (e.g. a single grain of sand) but the magnitude of the response may vary from tiny (e.g. single grain avalanches) to the size of the system (e.g. total sand pile avalanche).
4. The magnitude and frequency of these response events follows a negative power law distribution (with smaller events more abundant than larger events).
5. A mechanism for the system to dissipate energy.
6. The existence of many degrees of freedom within which internal processes can operate.

SOC can be a difficult and controversial behaviour to diagnose (Frigg, 2003). The key features listed above are necessary, but not sufficient conditions for the existence of SOC. Thus, the absence of these features proves the absence of SOC in a system, but their presence does not necessarily indicate that the system indeed exhibits SOC. Moreover, because sufficient conditions of SOC are not known, it is impossible to prove beyond doubt the existence of SOC in a system. Although they are not proof of SOC, each of the key features above can be used as "supporting evidence" for postulating SOC, with more pieces of supporting evidence permitting a stronger claim.

In many studies, the existence of a power law relationship between magnitude and frequency of events is the main tool to identify possible SOC behaviour, as it the easiest of the key features to identify. However, in absence of further arguments, the existence of a power law relationship is a weak basis for postulating that a system is exhibiting SOC. Many natural systems can provide us with power law relationships, for example Mandelbrot's fractal dimensions of coastlines (Mandelbrot, 1967), but these are not necessarily symptomatic of SOC. A further example is provided by the work of Rinaldo et al. (1993) and Rodríguez-Iturbe et al. (1994) where they claim to have found SOC in a numerical model of the development of drainage networks. However, these claims have been challenged by Sapozhnikov and Foufoula-Georgiou (1996) who question whether these models of drainage systems have enough degrees of freedom to organise themselves into a state whereby they may experience a critical change. Indeed, drainage systems, as those modelled by Rinaldo et al. (1993) and Rodríguez-Iturbe et al. (1994), are incisive and concentrative, partly from the model set up and flow routing algorithms, veering model results away from unstable solutions.

Hence, key to the identification of SOC is providing mechanisms for SOC to operate, i.e. explain *how* the power law relationship between magnitude and frequency of events may arise. Some studies inferring SOC show a direct mechanism for SOC within their models, for example the sand pile models of Bak et al. (1987, 1988), and the forest fire models of Drossel and Schwabl (1992). In general, providing a mechanism for a 'chain' of failures to propagate down part or whole of the system, is a key definition of SOC according to Bak et al. (1987, 1988).

Examples of SOC have been proposed in a wide range of natural systems ranging from earthquakes to forest fires (Bak, 1996). In fluvial systems, studies have found SOC using numerical, laboratory and field methods. Stølum (1996) identified SOC in a numerical model of river

meandering, whereby the sinuosity of the channel followed a power law distribution. The river would organise itself into a highly sinuous state, where one cutoff could occur, that in turn would increase local stream power and trigger a cascade of cutoffs in local bends. Then over time, the newly straightened channel would organise itself back into a tortuously meandered state and the process would occur again. This behaviour was previously noted in real river systems by Hooke (2003).

Fonstad and Marcus (2003) explored examples of SOC in lateral erosion, and river width adjustment. Stating that rivers are open dynamic energy dispersive systems, they modified the sandpile model (Bak et al., 1987) to produce a simple cellular model of bank erosion that showed that the magnitude of river bank landslips followed a power law frequency distribution. Furthermore, they surveyed natural bank erosion scars (indicative of lateral erosion) and showed that these too followed a power law magnitude–frequency distribution. Fonstad and Marcus (2003) also suggest a theoretical model for how river banks may organise themselves into a critical state whereby one failure deposits sediment within the channel that (may) then lead to another failure downstream.

Sapozhnikov and Foufoula-Georgiou (1997) also found evidence for SOC in laboratory models of braided rivers, with power law distributions in the channel dynamics. Furthermore, they suggest that braided rivers have many of the attributes required for SOC systems with large degrees of freedom and a high level of interconnectedness. Sapozhnikov and Foufoula-Georgiou (1996) also suggest that SOC systems require co-operative behaviour, so that any part of the system in the critical state can 'feel' changes in other parts. Sidorchuk (2006) used numerical models of gulley formation to show that early active phases of gulley development showed strong SOC through a power law frequency magnitude relationship, despite being driven by a constant discharge. But this relationship was far less strong during later stable periods, suggesting that evidence of SOC could be used to show when gulley systems were active or not. Sidorchuk (2006) also tested the model using variable discharge data (following a power law distribution) and found that unfortunately this masked this response, so that even during 'stable' periods, the model exhibited SOC.

If fluvial systems can exhibit SOC, then why is this important? In their introduction to the role of SOC in lake sediment deposition, Dearing and Zolitschka (1999) suggested that SOC is important for four reasons.

1. The variability in the output from a SOC system is not dependant on changes in energy inputs. In the sandpile example, the magnitude of the avalanche varies greatly but the input (one grain of sand) remains the same.
2. This can create a paradox, as extreme external inputs of energy (e.g. the input of many grains of sand at once) may disrupt the system in the short term, but equally large disruptions may be caused as a result of internal processes (as above).
3. The result of 1 and 2, is that to all intents and purposes SOC systems are unpredictable (though over time they may be represented statistically).
4. Therefore analysis of data from SOC systems may simply be describing the internal dynamics and instabilities, rather than the results of any external forcings.

Table 1
Sources of non linearity in natural systems and representation in the CAESAR landscape evolution model

Sources of non linearity	Examples in geomorphological systems	Representation
Spatial heterogeneity	−Spatial distributions of sediment, vegetation, soil moisture, …	Direct/indirect
Thresholds	−Sediment entrainment processes	Direct
Memory	−Alluvial sediment storage −Antecedent soil moisture	Indirect
Saturation	−Capacity limited sediment entrainment	Direct
Depletion	−Supply limited sediment entrainment	
Positive feedback	−Vegetative bank stabilization (vegetation is more likely to establish on non-eroding banks, causing these banks to be even more resistant to erosion)	Indirect
Negative feedback	−Bed armouring (caused by selective entrainment of smaller sediments and reducing further entrainment of smaller sediments) −Floodplain accretion (becomes more difficult as the floodplain accretes)	Indirect
Competition between processes	−Precipitation can infiltrate, evaporate or flow as overland runoff	Direct/indirect
Multiple modes of adjustment	−Lateral erosion vs channel incision in response to increased stream power −Response to sudden input of sediment	Indirect
Hysteresis	−Sediment entrainment on rising and falling limb of a flood	Indirect
'Random' variation in processes	−Turbulent sweeps in flow	Not
External forcing	−Climate, weather −Tectonics −Anthropogenic events	Direct

These implications also ring true for fluvial systems, in that river behaviour may not be predictable, and that little relationships may exist between system inputs and outputs. Point 4 is especially pertinent for palaeo studies of river systems, whereby attempts are made to link river behaviour to external forcings, such as previous climate changes (e.g. Coulthard et al., 2005). Indeed, this may go some way to explaining why we have such difficulties in identifying causality in longer term response in fluvial systems to forcings (e.g. Vandenberghe, 2003). Fonstad and Marcus (2003) also argue that for river bank systems this indicates that local failures are unpredictable, and that for SOC systems we have to consider the system as a whole instead of its components in isolation. This view, if correct, has important ramifications for the way we study and interpret river systems, especially for reductionist approaches.

2. Rationale

Here we have provided examples where non linearity and possibly SOC have been identified in fluvial systems, but what are the causes? Phillips (2003) reviews the sources of non linearity in geomorphic systems. These we have summarised in Table 1, along with two extra categories that we have identified and examples from fluvial geomorphology. However, close examination reveals that very few studies have been attempted to precisely identify the causes of non linearity. Many refer to 'erosional thresholds' or 'positive feedback mechanisms', which may well be true, but much of this causality is determined from interpretation and is often presented with little supporting evidence.

This leads to a major knowledge gap and raises important research questions. We have a good idea of what *may* cause non linearity and SOC in river systems, but we have very little firm evidence of precisely what causes it. For example, we know that armouring effects at the bed of a river can produce threshold effects, but we do not know how significantly this may affect the sediment yield. Is it the main cause of variability in sediment fluxes, or does it arise from sediment inputs from bank erosion processes (for example)? Non linear processes are very important in the operation and development of fluvial systems, yet we have remarkably little idea of what causes them, and how important the causes or triggers are.

In this paper we aim to determine some of the causes of non linearity in river systems, how significant they are, and whether or not river systems may exhibit SOC. We deliberately choose to focus on the causes of non linearity in the relationship between water and sediment discharge,

as this is fundamental to many other fluvial processes and landform development. Furthermore, comparatively much data exists on sediment yields and it provides a tractable metric. To investigate this, one possibility is to use experimental flume models that provide us with a high degree of control and ability to measure key variables (e.g. flow, topography). But even within a highly controlled laboratory environment, it is difficult to remove factors that may be influential, such as small variations in the initial conditions of the model or minor fluctuations in inputs such as water and sediment. An alternative, which gives us total control over our experimental environment (albeit at a reduced representation of some variables) is the use of numerical models. In particular, cellular automaton models are ideally suited for modelling spatially variable non linear phenomena.

Cellular automaton models are spatially explicit, using a regular grid of cells where simple universal rules represent the essential aspects of local processes. The rules, which can be either deterministic or stochastic, are universal, in the sense that they are equally applicable to all cells on the grid. They are local, in the sense that they define how each cell interacts with its immediate neighbours. Large-scale spatial connectivity arises from temporally iterative application of these local rules, and the resulting interactions, whereby any perturbations can propagate through the system. Hence, large-scale complex behaviour can emerge from the local rules. In the context of fluvial geomorphology, cellular automaton models have been applied to simulate braided river systems (e.g. Murray and Paola, 1994, 2003; Thomas and Nicholas, 2002), bank erosion (e.g. Fonstad and Marcus, 2003), river meandering (e.g. Coulthard and Van De Wiel, 2006), sediment yield (e.g. De Boer, 2001), alluvial fan development (e.g. Coulthard et al., 2002; Nicholas and Quine, in press), and fluvially driven catchment-scale landscape evolution (e.g. Coulthard et al., 1998; Luo, 2001; Crave and Davy, 2001; Coulthard and Macklin, 2003; Van De Wiel et al., in press). In the context of self-organized criticality, Bak et al. (1987, 1988) used a cellular automaton model to demonstrate SOC in cascading sandpiles.

In this paper, we use the cellular landscape model, CAESAR (Coulthard et al., 2000; Coulthard et al., 2002; Van de Wiel et al., in press), to determine what the relative effect or role of some of these factors are in generating non linear responses. This will directly test the impact of bed armouring, sediment supply (through landslips), historical contingency (through the sequencing of different storm events), and autogenic processes (including catchment shape).

3. Method

The CAESAR landscape evolution model focuses strongly on fluvial processes. Its suitability and a description of its operation have been described in depth within previous publications (Coulthard et al., 2000; Coulthard et al., 2002; Willgoose 2005; Van de Wiel et al., in press), but here we provide a précis of its operation. CAESAR divides a study catchment (or reach) into regular grid cells, whose values represent the elevation of the corresponding land surface. Over this gridded surface, a sequence of rainfall can be applied, and a hydrological model (an adaptation of Topmodel; Beven and Kirkby, 1979) is used to calculate whether sufficient moisture exists to generate overland flow. If overland flow occurs, surface water is routed across the cells using a multiple flow routing algorithm (see Coulthard et al., 2002; Van de Wiel et al., in press) that calculates a flow depth for each cell. This flow depth is then multiplied by the slope between neighbouring (down slope) grid cells to calculate a shear stress. This is used to calculate the volume of sediment eroded from a cell and transported to its downstream neighbours. To calculate the volume eroded, CAESAR uses the Einstein (1950) or Wilcock and Crowe (2003) sediment transport equation. Importantly, multiple (9) grain sizes are used and volumes eroded and deposited are the sum of the volumes moved from each different grain size. These multiple grain sizes are modelled within a sophisticated 3 dimensional representation of the stream bed and sub-surface that incorporates a series of 'active layers' (Van de Wiel et al., in press). These active layers are used to store the varying proportions of sediment of different sizes at different depths, allowing the model to account for the deposition and storage of coarse or fine sediment from previous events as a river bed or floodplain stratigraphy. This method allows the simulation of the effects associated with heterogeneous bedload mixes, such as the development of armoured layers, and selective transport and deposition. In turn, this permits CAESAR to model the threshold effects associated with these processes. Coupled with the multiple direction flow routing algorithm, which also allows the width of a channel to be simulated by more than one cell, this is a sophisticated 2d (3d with stratigraphy) representation of fluvial erosional and depositional processes. Slope processes are also represented, with a simple diffusion based model of soil creep, and a threshold based landslide model, whereby when a slope becomes steeper than a threshold angle, material is moved downslope from cell to cell until the slope is less than the threshold.

This detailed fluvial erosion and deposition model is the main reason why CAESAR is used in this study,

Table 2
Configuration of the simulation runs

Run	Catchment	Landslides	Rainfall event[a,b]	Sediment size[c]
1	A	Yes	10 mm	Mixed
2	A	Yes	10 mm	4 mm
3	A	Yes	20 mm	Mixed
4	A	Yes	20 mm	4 mm
5	A	Yes	30 mm	Mixed
6	A	Yes	30 mm	4 mm
7	A	No	10 mm	Mixed
8	A	No	10 mm	4 mm
9	A	No	20 mm	Mixed
10	A	No	20 mm	4 mm
11	A	Yes	Variable	Mixed
12	A	Yes	Variable	4 mm
13	B	Yes	10 mm	Mixed
14	B	Yes	10 mm	4 mm
15	B	Yes	20 mm	Mixed
16	B	Yes	20 mm	4 mm
18	B	Yes	30 mm	4 mm
19	B	No	10 mm	Mixed
20	B	No	10 mm	4 mm
21	B	No	20 mm	Mixed
22	B	No	20 mm	4 mm
23	B	Yes	variable	Mixed
24	B	Yes	variable	4 mm

[a] Rainfall events have a duration of 1 h and occur once each day.
[b] Variable rainfall events range between 10 mm/h and 30 mm/h.
[c] Mixed sediment grain sizes range between 1 mm and 256 mm.

along with its previously proven capability to demonstrate non linear behaviour (Coulthard et al., 1998), and the authors' familiarity with the model. Also, the model is fully deterministic. No stochastic rules are applied, so any variations in output between different simulations can only result from variations in inputs and the resulting variations in internal dynamics arising from the deterministic process rules.

4. Model set up

A series of numerical experiments were devised to test the relative importance on generating non linear responses and SOC of: (1) storm magnitude; (2) bed armouring; (3) sediment supply (through landslips); (4) the impact of variable storm events (including the impacts of historical contingency); and (5) catchment morphology (which includes historical contingency and autogenic processes).

These experiments were split into two groups (see Table 2). The first group, consisting of runs 1 to 12, focussed on magnitude, armouring, landslips and historical contingency, and used a very simple catchment shape (catchment A; Fig. 1A) to reduce any topographic effects to a minimum. On this simple
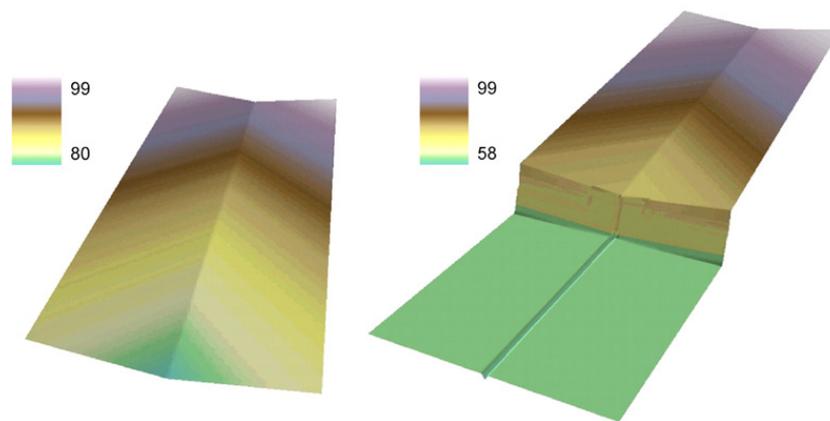
Fig. 1. Catchments used in the simulations. A. Simple catchment used in runs 1 to 12. B. Catchment used in runs 13 to 24. Catchment A is 200 by 100, 1 m diameter grid cells. Catchment B is 300 by 100, 1 m grid cells.

catchment, one hour of rainfall was applied at the beginning of every 24 h of simulated time. Three rainfall magnitudes were used: 10, 20 and 30 mm/h. For each rainfall magnitude, separate runs were carried out with heterogeneous (0.5 mm–512 mm) and uniform (4 mm) sediment distributions, with landslides enabled (runs 1 to 6). Additionally, four simulations were performed with landslides disabled (runs 7 to 10), again comparing different rainfalls and different sediment distributions. This matrix of runs allowed a direct comparison between the factors of armouring, landslips and storm magnitude. Finally, two variable climate runs were carried out (runs 11 and 12), with mixed bedload and 4 mm bedload, respectively. In these simulations storm size varied randomly between 10 and 30 mm/h. The duration of each of these twelve runs was 25,000 days or 25,000 identical events.

The second group (runs 13 to 24; Table 2) were identical to the first, but the morphology of the catchment was altered to include a step or section of valley floor at the base of the same small catchment (catchment B; Fig. 1B). Thus, during model operation, a small alluvial fan is sometimes created at the base of the main catchment. This catchment shape is designed with the purpose to create a single area of storage, which can allow us to determine what impact this may have on sediment yield from the catchment.

All sediment was treated as bedload. Sediment yields were measured at the water and sediment outflow point at the base of the catchment.

5. Results

The results for the first series of simulations (runs 1 to 12) are presented in Fig. 2, where the temporal variation in daily sediment discharge is shown for each

run. All show similar characteristics, with sediment yield starting high and gradually declining, and beginning to stabilise after 5000 to 10,000 days. This initial peak and tail off is caused by the catchment shape adjusting. The long profile of the catchment is initially linear, and during the course of a simulation it steepens at the top end because of erosion, and shallows in gradient at the lower end because of less erosion and some deposition. For the first days of simulation this results in comparatively steep gradients throughout, generating high yields of sediment. As material is eroded from the catchment, gradients lower, thereby reducing energy to erode, and consequently sediment yields decrease. This type of response is also found in other numerical simulations of river catchments and gully systems (Coulthard et al., 2005; Sidorchuk, 2006).

Fig. 3A shows this decline in more detail for run 1 (10 mm/h rainfall events, mixed bedload, landslides enabled). On average, the decay of sediment yield appears smooth, but this conceals some variation. A detailed look at 1000 events (days 9000 to 10,000; inset in Fig. 3A) demonstrates that some variability occurs, with sediment yields ranging from 0.75 to 1.4 m^3 per event. The magnitude–frequency relation of these sediment discharges follows a power law distribution (Fig. 4). Considering a similar scenario, but with homogeneous 4 mm sediment sizes run 2, (Fig. 2) clearly shows a far smoother line describing sediment yield, with less variability. This indicates that the mixed bedload has an important role in increasing non linearity. Increasing rainfall sizes to 20 and 30 mm/ h (runs 3 to 6; Fig. 2) show a similar pattern: variability for simulations in homogeneous sediment (runs 4 and 6) is less than the variability in runs with heterogeneous sediments (3 and 5). However, the difference is less distinct as rainfall increases. Additionally, variability in
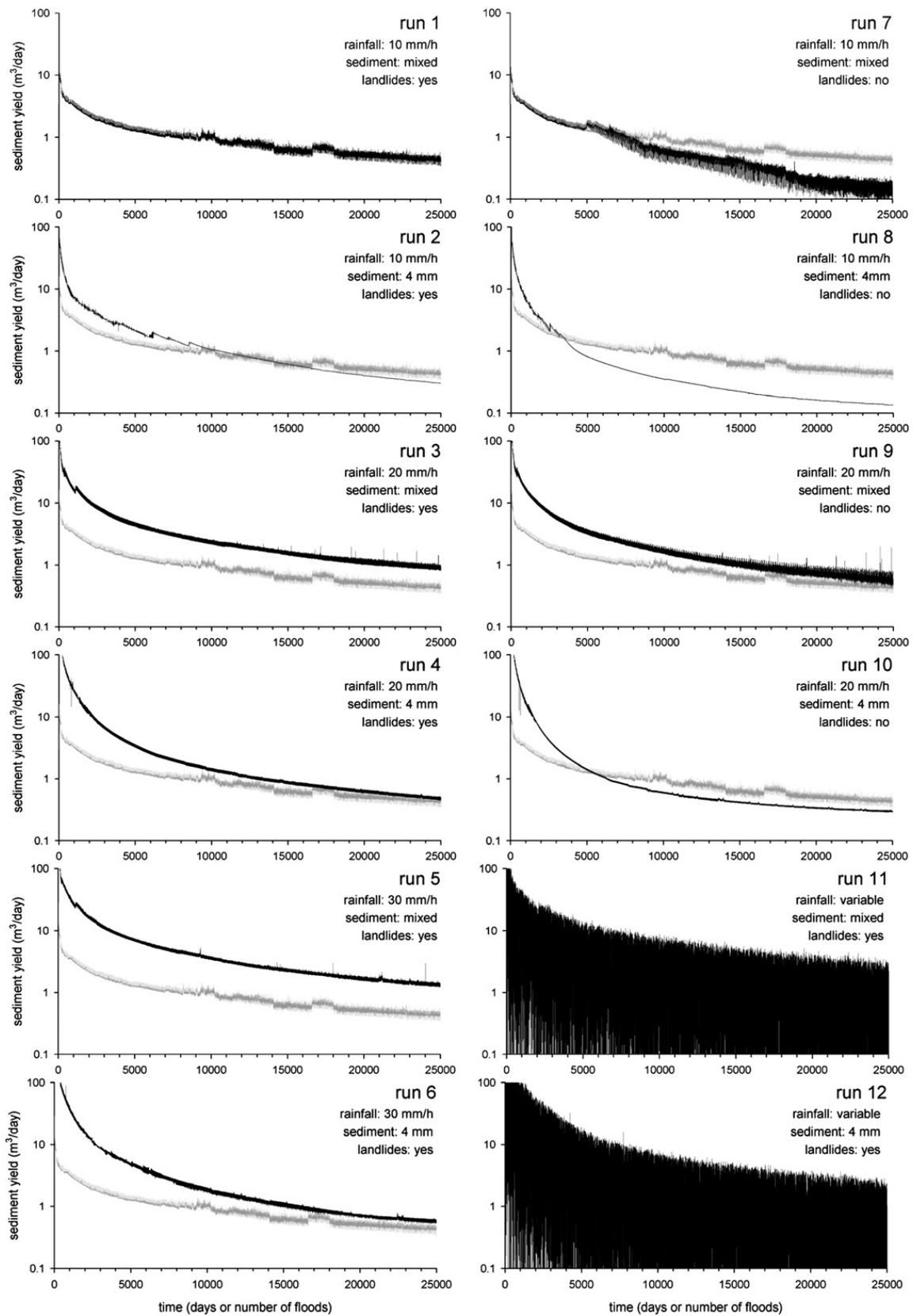
Fig. 2. Sediment discharges for runs 1 to 12 (catchment A). Sediment discharges for run 1 are repeated on each graph (in gray) for reference.
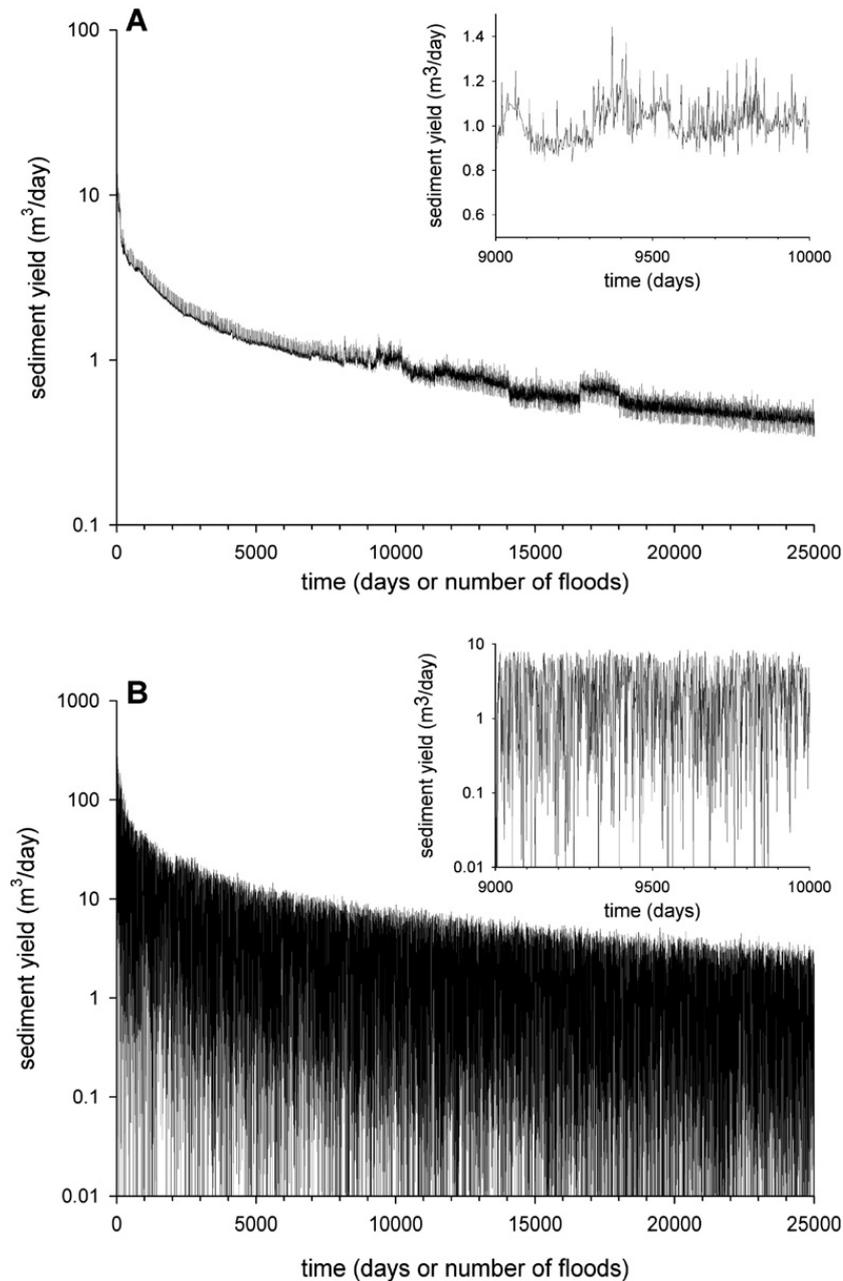
Fig. 3. Sediment discharges for run 1 (A) and run 11 (B). Insets shows detail for days 9000 to 10,000. Note the different vertical scales in the figures.

the mixed bedload runs decreases with higher rainfall events (*cf.* runs 1, 3 and 5; Fig. 2). A final observation is that sediment yields in these simulations are consistently higher than those during the 10 mm/h rainfall runs, which is to be expected given larger flood sizes.

Disabling the landslide algorithm (runs 7 to 10; Fig. 2) allows channels to incise without the over-steepening of the banks, which can cause material from slopes to be added. These show a similar pattern to simulations carried out with landslides, in that the mixed bedload simulations (runs 7 and 9) have more variation in bedload than comparable simulations with homogeneous 4 mm bed material (runs 8 and 10). However, in comparison to runs with landslides, substantially more variation is observed in both 10 and 20 mm/h rainfall mixed bedload examples. Within CAESAR, landslides can provide a supply of sediment other than by the mining material from the river bed. Therefore, this would imply that the addition of material from slopes can reduce the variability of sediment yield, smoothing the irregular response of the bed. Interestingly, when landslides are disabled, the decay in sediment yield is far more rapid than for the landslide inclusive runs, indicating how the input of slope processes may maintain the delivery of sediment.
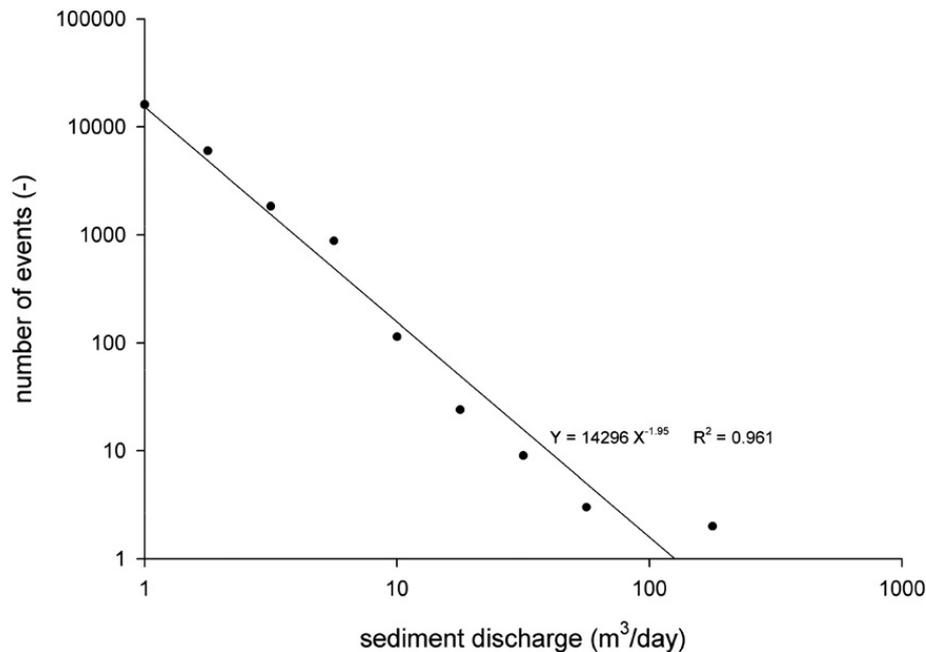
Fig. 4. Magnitude–frequency relationship for sediment discharge, from run 1.

The variable climate simulations (runs 11 and 12; Fig. 2), with rainfall randomly varying between 10 and 30 mm/h, produce a very different response. Here, widespread variability occurs in the sediment yield, though a general decrease in the upper limit of sediment yield is observed (broadly similar to the decay in sediment production shown in all the previous runs). This is shown in more detail in Fig. 3B, revealing the full extent of the variability with sediment yields varying over four or more orders of magnitude, which is significantly different from the results with steady rainfall magnitudes (runs 1 to 10; Figs. 2 and 3). Sediment yields are also at times far higher than those observed in the steady rainfall simulations despite the maximum rainfall being the same.

The second group of simulations using the different catchment shape (Fig. 1B) showed markedly different responses (Fig. 5). The sediment yield time series for run 13 (10 mm/h rainfall, mixed bedload, landslides enabled) shows three distinct phases (Fig. 6). During the first 2500 days, sediment yields are low, almost negligible but with variability. From 2500 to *c*.11,000 days significant variability occurs with sediment yields ranging from 0.5 to 100 m^3/day. These sediment yields are very high, compared with similar conditions applied on the simpler catchment (run 1; Fig. 3A), where the maximum was less than 10 m^3/day and the mean 1 m^3/day. The third phase, from 11,000 to 25,000 days, is closer to the results seen by the previous group of simulations, with a comparatively steady sediment yield.

These three phases can be linked to the geomorphology as shown in Fig. 7. Phase 1 corresponds to a period where an initial alluvial fan is being built (Fig. 7A) resulting in low sediment yields, a disconnected state. Phase 2 corresponds to a period of fan re-working (Fig. 7B and C) as well as extensive incision of the main channel above the fan (compare circled area in Fig. 7B with corresponding area in Fig. 7A). During this phase, the data for sediment yield (Fig. 6) show that large volumes of sediment are released, as well as there being considerable variability. This is a period of relatively intense geomorphic activity, with interactions between main channel incision and alluvial fan development, and where the system could be described as semi-connected. After 11,000 days, we enter phase 3, where a stable channel has been cut through the fan, effectively bypassing the storage component of the fan and directly connecting the catchment to the outlet (Fig. 7D). This is apparent in Fig. 7C and D, where little change occurs, and in Fig. 7D, where the small stable channel running across the top of the fan is apparent. It has entered a connected state and is operating much as it had done in simple catchment simulations, explaining the reduction in sediment yield variability. This transition and shift in fan behaviour is quite apparent in an animated sequence of these graphics that can be viewed at http://www.coulthard.org.uk/downloads/animations.htm.

The results from the other steady climate simulations on catchment B (runs 14 to 22; Fig. 5), combining different (but constant) rates of rainfall, bedload mixes
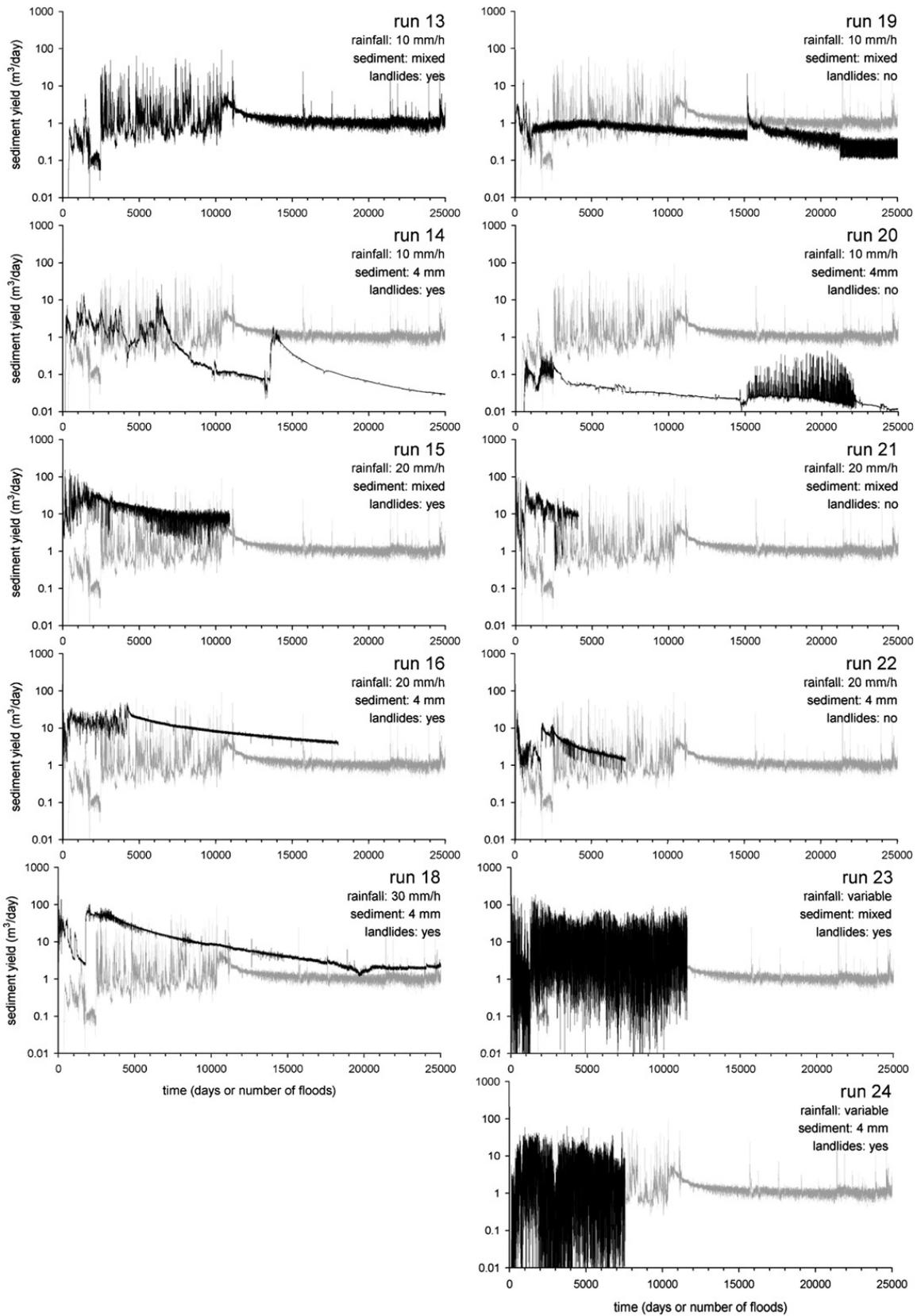
Fig. 5. Sediment discharges for runs 13 to 24 (catchment B). Sediment discharges for run 13 are repeated on each graph (in gray) for reference.
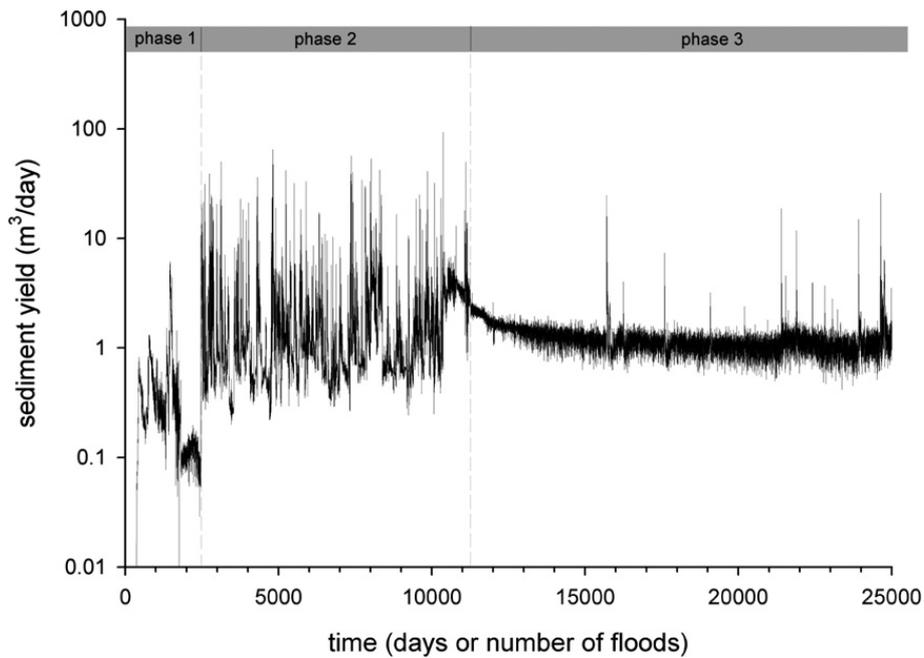
Fig. 6. Phases in sediment discharge for run 13. See text for explanation.

and whether or not landslides are enabled, show that a wide variety of behaviour exists. Nonetheless, similarities can be drawn between the disconnected (fan construction), re-working, or connected states as described with Figs. 6 and 7, and the varying levels of variability previously linked to different bedload and landslide configurations. For example, run 20 (10 mm/h rainfall, uniform sediment, landslides disabled) shows a clear initial period of disconnection, then re-working, then connected, as per Fig. 6, but here the variability
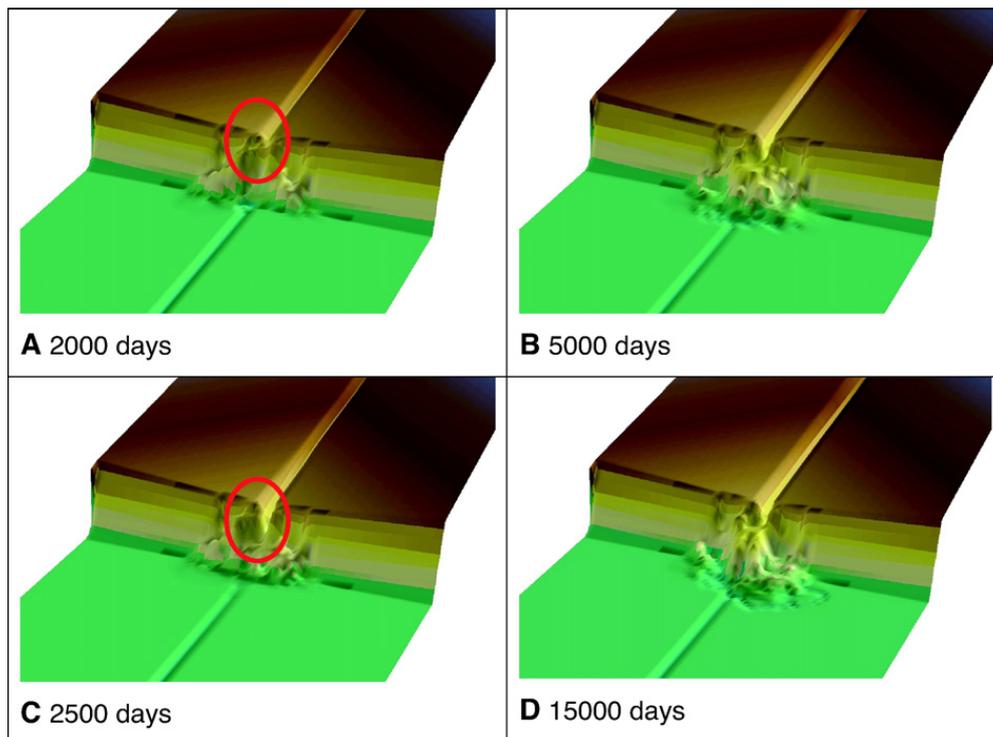

Fig. 7. Evolution of fan topography after 2000 days (A), 2500 days (B), 5000 days (C) and 15,000 days (D).

during the connected state is much less, because of the single grain size bedload. However, this run is also interesting: at *c.*15,000 days it becomes unstable again, entering another period of re-working. Such behaviour is apparent in many of the examples shown in Fig. 5, and is indicative of how the alluvial fan system may be stable – or connected – for quite some period, then suddenly change into a period of re-working resulting in significant variability. Some of the runs detailed here did not complete the full 25,000 days of simulation because of the long run times associated with the more complex topography, especially when combined with increased storm magnitudes.

Similar to runs 11 and 12 on catchment A, the variable climate simulations for catchment B (runs 23 and 24; Fig. 5) show a very high variability in sediment yield. However, unlike the simple catchment runs, larger scale variations are exhibited in the magnitude of response. This is shown in more detail for the mixed sediment simulation (run 23) in Fig. 5 (compare with Fig. 3B). Again, this largely results from changes in the formation of the alluvial fan and the connectivity of the system.

6. Discussion

In the Method section, we aimed to explore how five different factors may influence non linearity. We shall discuss each of these in turn and how they may operate in combination and finally how these may give rise to SOC behaviour.

6.1. Non linearity

6.1.1. Magnitude of regular storm events

Changes in the magnitude of regular storm events seem to have little effect on non linearity. Increases in storm magnitude (10, 20 and 30 mm/h) increase the volume of sediment per given event, as would be expected. However, with increasing storm size a reduction in variability occurs for the mixed grain size simulations. This may result from the increased size of the flood being powerful enough to erode all material, whereas the more moderate floods from the 10 mm/h runs operating close to or below the erosion thresholds of some of the larger grain size material. However, there is a small increase in the variability of the 4 mm runs, which is significantly reduced when landslides are removed. This would imply that for larger floods, where the transport thresholds of the heterogeneous bedload are surpassed, the largest control on bedload yield is landslides.

6.1.2. Bed armouring

Considerably more variability occurs, across all runs, when CAESAR is set up with heterogeneous distribution of bed grain sizes. Much of this variability decreases or disappears when a single (4 mm) grain size is used. We suggest that the development of armour layers, and the non linear transport effects caused by the interactions of heterogeneous bed material are causing these variations. Over the course of the 25,000 floods, CAESAR will develop its own distribution of bed grain sizes, and within this distribution areas will occur with coarser material that will have higher erosional thresholds, because of the sediment transport law used. This spatial variability in surface grain size may change from flood to flood, giving a different yield of sediment sizes. Furthermore, because of the active layer system used within CAESAR, a coarser surface layer may protect or armour finer lower layers, so when this surface layer is eroded this may release a disproportionately large volume of more easily eroded sediment from below. As mentioned above, larger flood sizes reduce the variability, indicating a relationship may exist between the distribution of grain sizes and flood size that can lead to maximum variability. In these runs, the distribution of sediment grain sizes may not have had enough large sizes to resist the larger flood magnitudes, resulting in less variability in the larger floods.

6.1.3. Sediment supply

The sediment supply, through the inclusion or exclusion of landslides, showed two alternative reactions. Firstly, for mixed bed material, removing landslides increased variability and also reduced overall sediment yield with a greater decrease in sediment yield for individual floods. The introduction of landslides appears to decrease variability by dampening the signal from bed armouring. Without landslides, the only mechanism for variability is the bed armouring, and the regular addition of slope material through landslides appears to counteract this process slightly. This hypothesis is re-enforced by the second reaction, whereby for single grain size runs the removal of landslides reduces variability, implying that the largest control is landslides. This would appear to have a series of interesting inverse effects, whereby the reductions in irregular sediment supply from landslips result in an increase in the variability.

6.1.4. Variable storm events

Random rainfall events between 10 and 30 mm/h caused a massive increase in variability in comparison to the three above factors. We suggest that this is caused

by historical contingency, which here may be manifested in two ways. Firstly, through a limited memory in the hydrological model and, secondly, through a geomorphic memory. Concentrating initially on hydrological processes, the use of Topmodel means there is an element of antecedence within the catchment, where hydrological response to a rainfall event depends not only on the characteristics of the event itself, but also the relation to prior rainfall events. For example, a 30 mm/h rainfall event following a 30 mm/h rainfall event will produce a larger flood than a 30 mm/h rainfall event

following a 10 mm/h rainfall event. To test whether this has a significant effect on sediment yields, we compared frequency magnitude distributions of the *floods* generated by the rainfall events with frequency magnitude distributions of the sediment discharges (Fig. 8). A clear difference was observed between the flood magnitude (graph A) and sediment (graph B) outputs, so hydrological differences are not *directly* manifested in the sediment record.

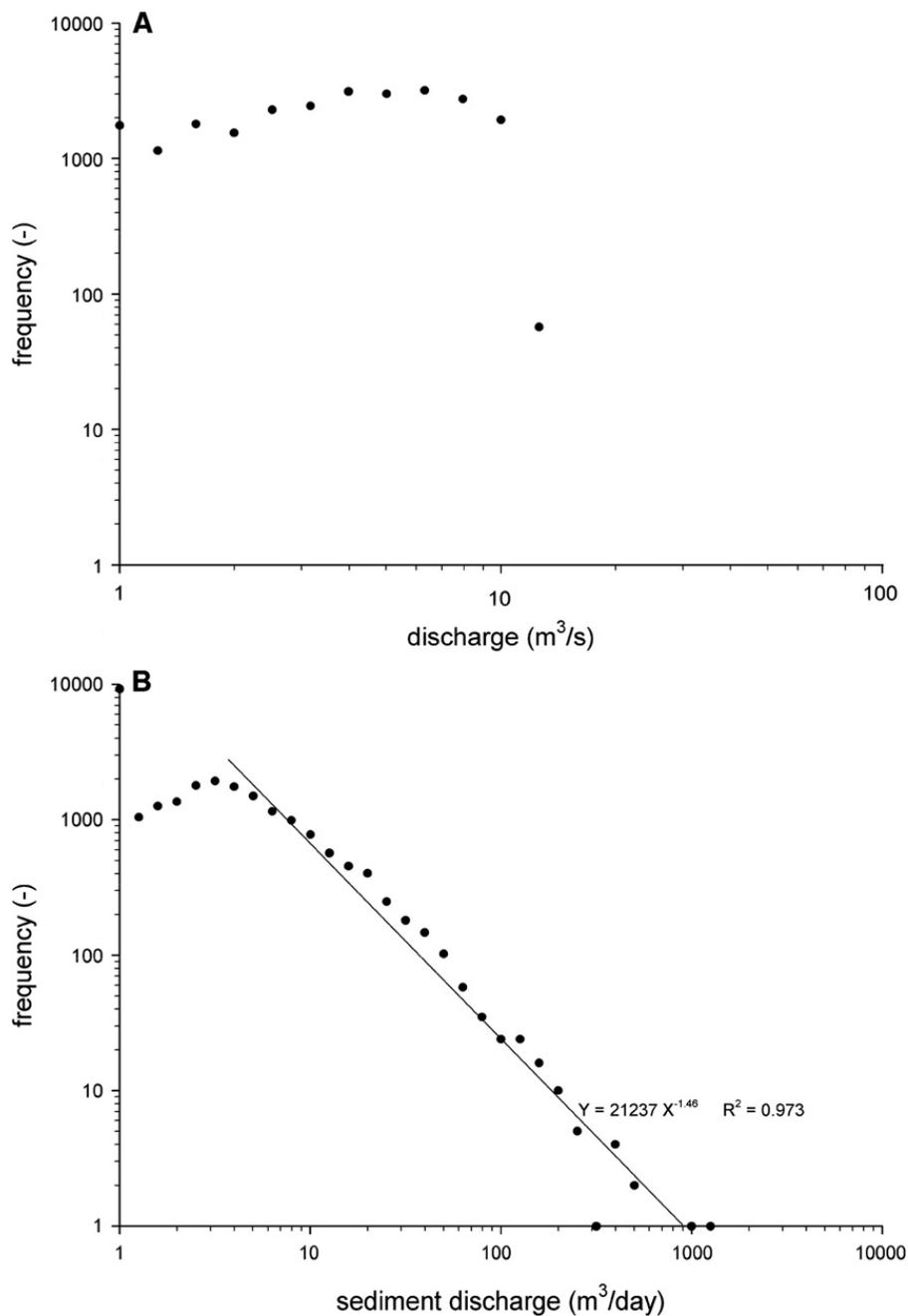These experiments have been deliberately set up to be parsimonious in the representation of catchment

Fig. 8. Magnitude–frequency relationships for flow (A) and sediment discharge (B).

processes. Therefore, the only other logical explanations for the disparity between water and sediment yields are geomorphological or sedimentological factors, such as the storage and re-mobilisation of sediment, *i.e.* a geomorphic memory. Even within such a simple basin, it would appear that there is significant scope for these effects to be important. For an explanation, consider the scenario where the catchment experiences four consecutive large floods. We may see a diminishing sediment yield for each of the four events, because the previous event has removed larger quantities of sediment, therefore reducing sediment availability. If we consider the scenario where we have a series of three small floods, followed by a large flood, the sediment yield for the final large flood may be far higher than that of any of the four large floods in the previous paragraph. This results from greater supply or availability of sediment. In effect, the smaller floods have moved or prepared the volumes of sediment ready to be evacuated by the large flood when it comes. Therefore, we argue that the geomorphic factors, or the storage, and subsequent re-working of sediment (in effect the geomorphic history of the catchment) has a 2–3 orders of magnitude greater impact on variability than the previous three factors.

6.1.5. Catchment morphology

Catchment morphology appears to have the strongest control on the levels of non linearity for all simulations. The addition of an area where sediment can be stored and re-mobilised developing an alluvial fan, creates great potential and several degrees of freedom for non linearity. This is clearly illustrated by comparing runs 1 and 13 (Figs. 3A and 6), which are identical apart from the shape of the catchment. Here, variability operates over two orders of magnitude more and also generates total yields up to 10 times greater. Again, this could be viewed as historical contingency, as different shapes of catchments (which result from the catchments history of evolution) have large impacts on sediment yields. Similar reactions occur to changes from mixed to single grain sizes, and with the inclusion and exclusion of landslides. But major shifts in the patterns are also observed, and sudden changes from high to low variability that reflect different phases of activity in the small alluvial fan. In the final runs, with variable climate (e.g. run 23; Fig. 5), increases in variability occur, similar to those seen from the simple catchment runs (e.g. run11; Fig. 3B), but these are clearly superseded by the effects of catchment morphology.

Comparing these factors, a clear hierarchy exists, with storm magnitude (in isolation) and landslips having little impact on variability. The introduction of hetero-geneous bed material significantly increases variability, but this effect is small compared to how the catchment *processes* variable storm sizes. This distinction is important, because as Fig. 8 shows, it is not just the variable rainfall that causes the changes, but more how the catchment geomorphology interprets and processes that rainfall. Finally, catchment morphology or shape can clearly have a massive difference, 2–3 orders of magnitude greater than any of the above.

We interpret this as showing the effects of historical contingency. But even from these simple model set ups, there appear to be different scales of operation for historical contingency. The results from variable climate simulations show how 'short term' storage and re-mobilisation across a few flood events can lead to significant changes in sediment yield, yet Fig. 6 would indicate that, when looking at catchment morphology, historical contingency operates over much longer time scales. We have picked out three phases of operation over 25,000 floods for run 13 (Fig. 6), and similar phases of historical contingencies in larger scale fluctuations can be seen across all of the runs from catchment B (Fig. 5). We can summarise this in the conceptual model presented in Fig. 9, which shows three levels or scales of historical contingency: small scale, due to internal processes (e.g. supply from different flood events); medium scale, due to major morphological changes (e.g. fan changes); and large scale (here a reduction in sediment supply in a post glacial environment). By combining them we can create a signal similar to those from these simulations. Importantly, all these processes are autogenic, in that they are driven by internal factors, and whilst we have simplistically represented them as independent, they may (and probably will) interact with each other. Furthermore, there will be different temporal and spatial scales of historical contingency, which could be linked to the landform concerned. Here, we have illustrated this with a simple alluvial fan. The re-working of an alluvial terrace may, for example, operate over a far longer time scale and may have a more (or less) sudden impact than changes in a fan may have. Similarly, a shift from a braided to a meandering (or vice versa) may have a different magnitude and frequency of impact.

Nonetheless, analysis such as above may help us understand what factors are affecting catchment behaviour.

As indicated in Table 1, CAESAR can directly or indirectly address most sources of non linearity, as indeed we have tried to illustrate in our simulations. However, CAESAR does not simulate turbulence, which in real (*i.e.* non-virtual) catchments could be
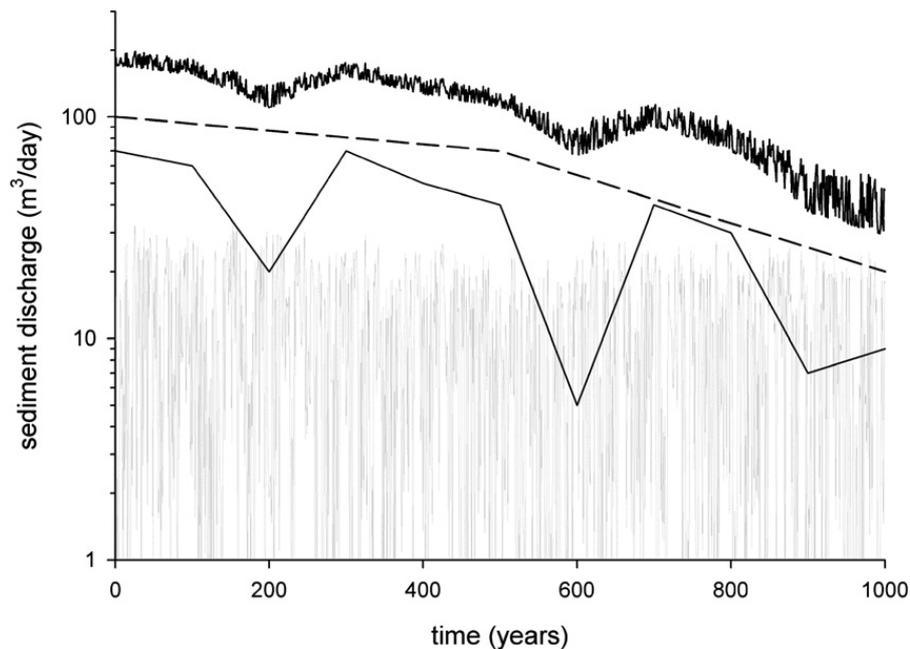
Fig. 9. Conceptual model of historical contingency on multiple levels: small scale internal processes (gray line), major morphological change (solid black line) and post glacial sediment supply (dashed black line). The cumulative effect of these is shown in the top line.

another source of non linearity. The apparent randomness of turbulent flow, *i.e.* the irregular magnitude and spatial distribution of turbulent sweeps and bursts, may cause irregularities in microscale bedload transport (e.g. Jackson, 1976; Sumer et al., 2003). This can cause irregular formation and distribution of pools, riffles, and dunes, which in turn may affect rates of bedload transport on a larger scale. Clearly, this could be a possible cause for the observed non linear variability of sediment yields in real catchment systems (e.g. Ashmore, 1988; Gomez and Phillips, 1999; Cudden and Hoey, 2003). Although our simulations offer no insight as to whether this indeed is the case, we can conclude that turbulence is not a necessary condition for non linear sediment yields. Rather, our simulations suggest that the autogenics of the catchment are sufficient to create non linearly variable sediment yields.

6.2. Self organised criticality

As mentioned in the Introduction, SOC can be a difficult and controversial behaviour to diagnose. Although power law relationships between magnitude and frequency of events are required to postulate SOC, they are by themselves insufficient to infer the existence SOC. Key to the identification of SOC is providing a mechanism for SOC to operate. Hence, we not only focus on the power law relationships between magnitude and frequency of sediment yield as an indicator of

SOC in catchment systems, but we also identify the nature of the critical state and suggest possible cascading processes or mechanisms by which the system can reach this critical state.

The existence of power laws relationships between magnitude and frequency can be seen in Fig. 4, and in Fig. 10, where sediment yield from the variable climate simulation (runs 11, 12, 23 and 24) is plotted. For catchment A (Fig. 10A) these simulations shows a very good negative power law relationship, although the relation does not cover the entire spectrum of sediment yields. In catchment A, the power law relation breaks down for very low sediment yields (<3 m^3/day). However, the power laws are very pronounced for larger sediment yields, which are geomorphologically significant. Importantly, this pattern does not directly relate to the climate signal, as discussed above, and is, therefore, a function of how the fluvial processes within the model operate and process the climate signal. This suggests strongly that these simulations, and, thus, by implication natural river systems, may be systems exhibiting SOC.

In both catchments slightly more scatter occurs around the power law with 4 mm grain sizes when compared with the mixed sediment sizes (respectively $r^2 = 0.96$ *vs.* $r^2 = 0.97$ and $r^2 = 0.96$ *vs.* $r^2 = 0.98$; Fig. 11). Moreover, the power law relations in simulations with constant rainfall (e.g. run 1; Fig. 4) show that climate variability is not a requirement for SOC variability in sediment yields. Hence, our
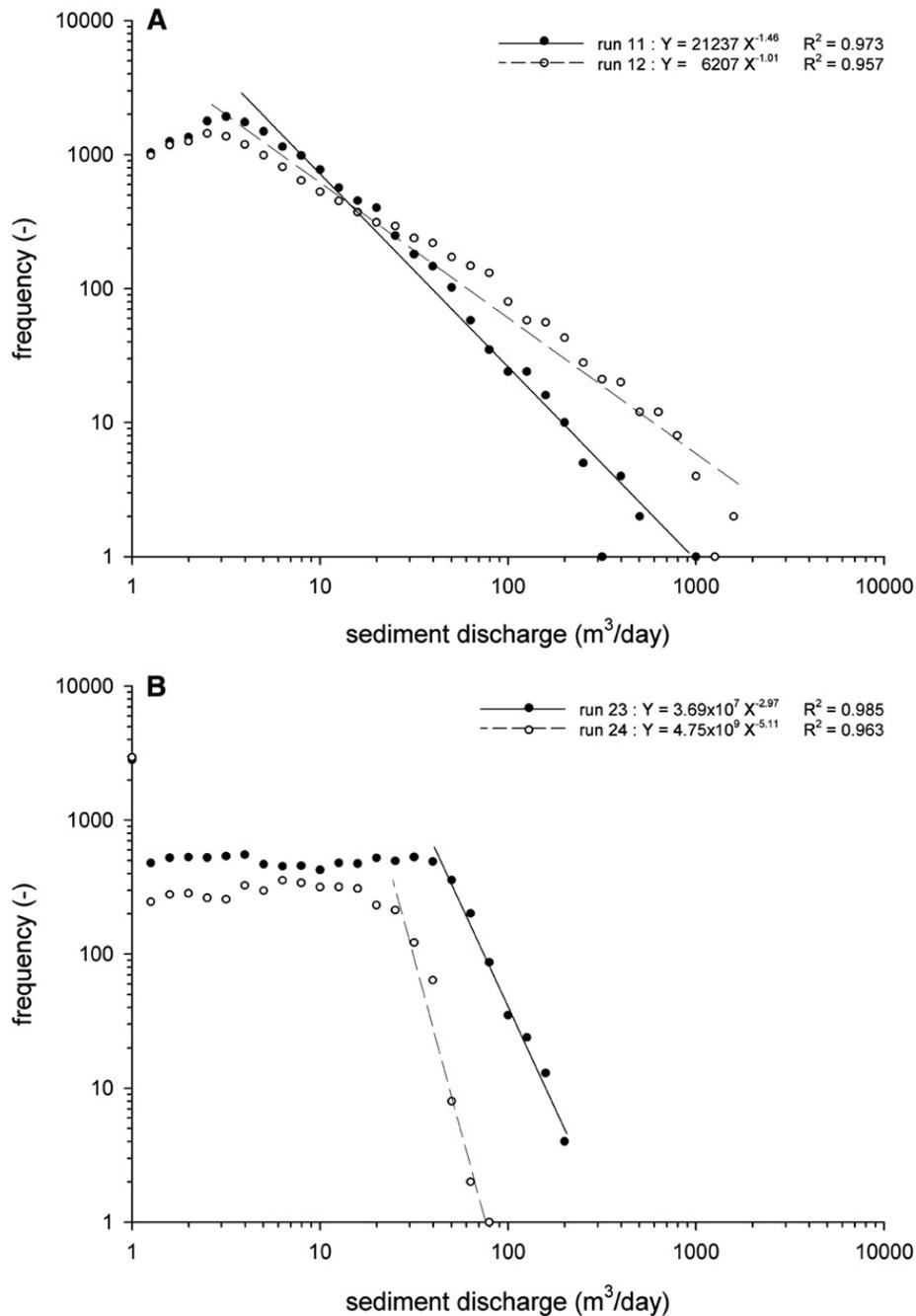
Fig. 10. Magnitude–frequency relationships for sediment discharge in variable climate simulations on catchment A (A) and on catchment B (B).

simulations suggest two possible sources for generating SOC behaviour in catchment systems: first, the interaction of a variable climate with erosion and sediment storage; and second, the impact of heterogeneous bed material on spatial and temporal differentiation in sediment storage.

We wish to present the following as a potential mechanism for the adjustments to occur within the simple domain of catchment A. Consider the simple long profile of the channel from catchment A. If we have incision at some point along the channel, this may cause

a wave of erosion migrating headwards — in effect movement of a mini-knick point. This migration is controlled, in its inception and propagation, by local variations in erosional thresholds (e.g. because of the heterogeneous bedload and subsequent armouring effects). This allows the 'cascade' response found in SOC systems, whereby the initial incision causes incision further up the channel network, and where the magnitude of that secondary incision can vary from a single computational cell to the entire network. A clear parallel exists between such a description and the one
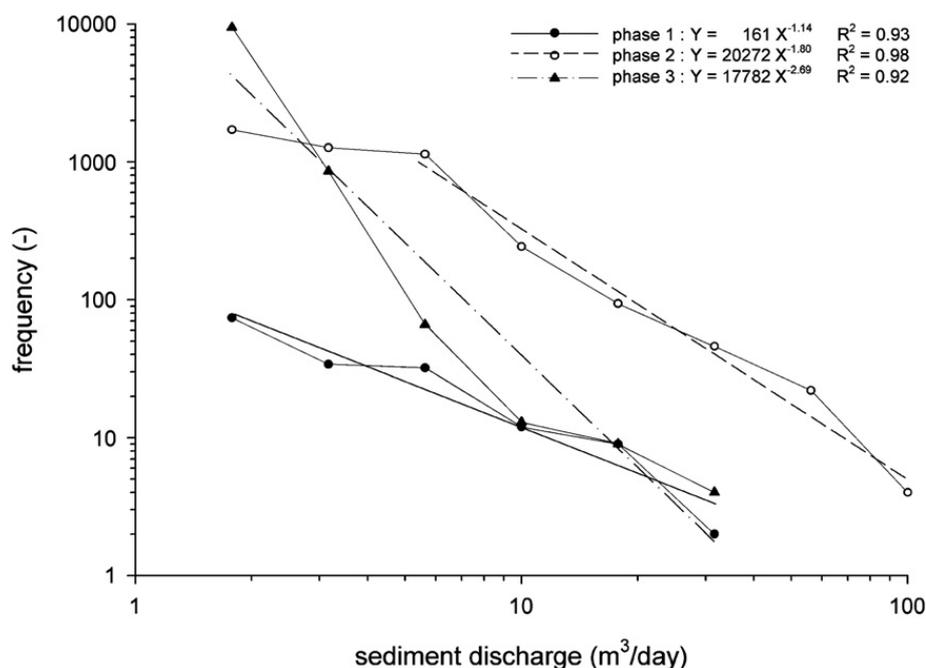
Fig. 11. Magnitude–frequency relationships for sediment discharge during each of the three phases of connectivity in run 13 (*cf.* Fig. 6).

dimensional slope where Bak et al. (1987, 1988) carried out their numerical experiments on landslides.

Catchment B shows a far poorer power law relationship with a variable climate (Fig. 10B), with small to medium sediment discharges following a even distribution and only larger events following a power law. This firstly results from the buffering properties of the fan, and secondly because these events are taken over 25,000 events, during which the fan morphology has evolved considerably. To illustrate this point, in previous sections we have described how three distinct phases of development occur in catchment B (Fig. 6), with different levels of variability during fan construction, re-working, and throughput. Here, we have carried out a magnitude–frequency analysis of these three phases of geomorphic activity. Each of the three phases shows a clear power law relationship (Fig. 11), although with varying degrees of scatter. In phase 1, when the fan is being built and the drainage network is disconnected from the outlet, power law relationship is relatively weaker ($r^2 = 0.93$). However, in phase 2, when the system moves to a semi-connected drainage network and the fan is being reworked, there is a stronger power law relationship ($r^2 = 0.98$). As the system moves to phase 3, when the drainage network is fully connected, the power law again weakens ($r^2 = 0.92$). More importantly, the slope of the regression power law relation varies between each of the three phases ($b = -1.14$, $b = -1.51$ and $b = -2.68$, respectively). Hence, a distinct difference exists in the distribution of sediment yield during these three phases. This suggests that the nature of the SOC, or the nature of the critical state depends on the connectivity of the drainage network. It also implies, that different to traditional SOC systems, the critical state of a catchment can vary in time, as sudden changes in drainage network connectivity may result in sudden changes in the SOC behavior of the system.

Therefore, we propose a second mechanism for SOC, founded on the observation that the SOC behavior of catchment B depends on the connectivity of the drainage network. Unlike catchment A, where lateral changes were restricted because of the narrow confines of the valley, here changes in the *plan form* configuration of the catchment and alluvial fan, in particular, affect the connectivity and, thus, the output. Any alteration in the network, for example through a sudden sediment delivery or the break-up of an armour layer (as per catchment A), as well as through *plan form* changes, may then cause a cascade of geomorphological adjustments, which can be of any magnitude. These adjustments cause corresponding variations in the sediment yield at the outlet. Imbedded within this response is the effect of heterogeneous bed material (as described for catchment A), but importantly, this second mechanism operates with a regular climate signal. It does not require a variable climate. The 'critical state' seems to occur when the catchment is semi-connected, *i.e.* when most geomorphic activity occurs. At this point, the catchment and fan combination has organized itself into a state where no (or minimal) net erosion exists at each point. It

has reached a point of dynamic equilibrium, with neither widespread channel incision nor fan aggradation.

Having proposed these two mechanisms, the next step for future studies of SOC using the CAESAR model is to further unpick these simulations. To back up these findings, we need to relate the time series of sediment yields to actual events within the catchment. This could involve measuring the number of cells eroded during an event to confirm the first proposed SOC mechanism, though it may be necessary to look at sub event time scales. Similarly, exact changes in channel plan form and connectivity may be related to spikes in the sediment output for catchment B.

In the Introduction, we discussed some of the implications of SOC; small inputs may give disproportionately large reactions; the systems are largely unpredictable and they have a high degree of internal dynamics. Here, we have demonstrated that simulations of morphologically very simple river catchments can generate highly non linear behaviour and possibly exhibit SOC. We believe that these simulations fulfill all of the 'key features' of SOC systems that we outlined in the introduction. The implications of this are not straightforward. It may be reasonable to state that SOC implies an invariance of process, so we cannot predict whole river system behaviour using at-a-point or empirical relationships (as suggested by Fonstad and Marcus, 2003). For example, we cannot describe how the system may geomorphologically respond to floods by studying just a single reach or sub catchment (Coulthard et al., 2005). Furthermore, we could also state that rivers are highly unpredictable. This raises important implications for river managers, and scientists examining or interpreting river behaviour, as a short period of fluvial activity may not be indicative of the system as a whole. Possibly the most important implication is that small inputs can give a large response. How great are these internal or autogenic changes in relation to external changes or forcings? This is a very important question to answer, as it may well be that the impact of climate change is less than the natural internal variability within a system. Alternatively, climate change can introduce changes in the system, which when processed in the catchment can result in significant non linear variations in sediment yield. Both scenarios have important implications for managing rivers in a changing climate, and for the interpretation of previous climates on rivers. Another fascinating facet of SOC is that these simple catchment systems have organised themselves towards this critical point. Are concepts such as the graded river (Mackin, 1948) really reflections of how river systems may (in certain conditions) organise themselves into a critical

state? Have our model catchments configured themselves to expend a minimum amount of energy to transport sediment from them given the non linear processes acting within them? Is the concept of dynamic equilibrium describing a system in a state of self organised criticality?

7. Conclusions

The results presented here have shown that even for very simple catchment configurations, large amounts of variability in sediment discharge can be caused through autogenic factors such as bed armouring, topographic changes and sediment storage and re-mobilisation. These effects also considerably change how a catchment processes external forcings such as changes in rainfall. Furthermore, considerable evidence exists that the systems show SOC behaviour, which may have important implications for understanding how fluvial systems evolve.

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